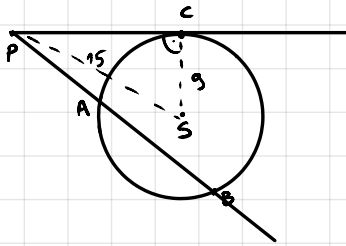


23, 24, 32, 35, 40

23.



$$|PC|^2 + 9^2 = 15^2$$

$$|PC|^2 + 81 = 225$$

$$|PC|^2 = 144 \wedge |PC| > 0$$

$$|PC| = 12$$

$$\frac{|PA|}{|AB|} = \frac{1}{3}$$

$$3|PA| = |AB|$$

Wynika z twierdzenia o stycznej i siecznej

$$12^2 = |PA| \cdot |PB|$$

$$144 = |PA| \cdot (|PA| + 3|PA|)$$

$$144 = 4|PA|^2$$

$$|PA|^2 = 36$$

$$|PA| = 6$$

$$|AB| = 3 \cdot 6$$

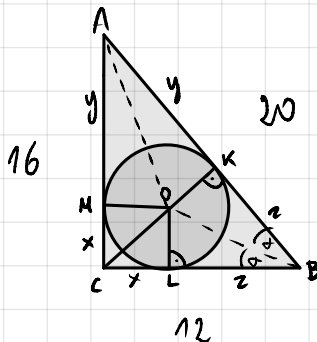
$$|AB| = 18 \text{ cm}$$

24.

$$16^2 + 12^2 = 256 + 144 = 400 = 20^2$$

$\triangle ABC$ jest prostokątny

a)



$\triangle OCL$ jest przystający do $\triangle MOC$ (BKB)

$$|MC| = |CL| = x$$

$\triangle AOM$ jest przystający do $\triangle AKO$ (BKB)

$$|AM| = |AK| = y$$

$\triangle KOB$ jest przystający do $\triangle BOL$ (BKB)

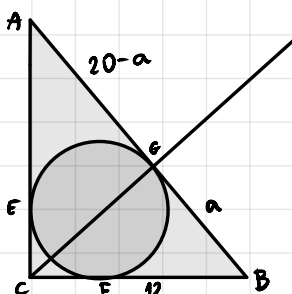
$$|KB| = |LB| = 2$$

$$\begin{cases} x+y=16 \\ x+z=12 \\ z+y=20 \end{cases} \Leftrightarrow \begin{cases} x+y=16 \\ z=12-x \\ z+y=20 \end{cases} \Leftrightarrow \begin{cases} x+y=16 \\ x=16-y \\ x=16-12=4 \end{cases} \wedge \begin{cases} 12-x+y=20 \\ 12-16+2y=20 \\ y=12 \end{cases}$$

$$z = 12 - x = 12 - 4 = 8$$

Bok AB został podzielony na odcinki o długościach 12 i 8, bok AC został podzielony na odcinki o długościach 12 i 4, a bok BC 4 i 8.

b)



$$\frac{a}{20-a} = \frac{12}{16}$$

$$16a = 12(20-a)$$

$$16a = 240 - 12a$$

$$28a = 240$$

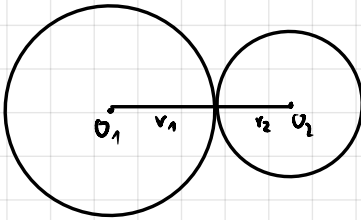
$$a = \frac{240}{28}$$

$$a = 8\frac{16}{28}$$

$$a = 8\frac{4}{7} \text{ cm}$$

$$20-a = 20 - 8\frac{4}{7} = 11\frac{3}{7} \text{ cm}$$

32.



a)

$$r_1 = 3a$$

$$r_2 = 4 - a$$

$$a \in (0, 4)$$

$$|O_1 O_2| = 8$$

$$|O_1 O_2| = r_1 + r_2$$

$$8 = 3a + 4 - a$$

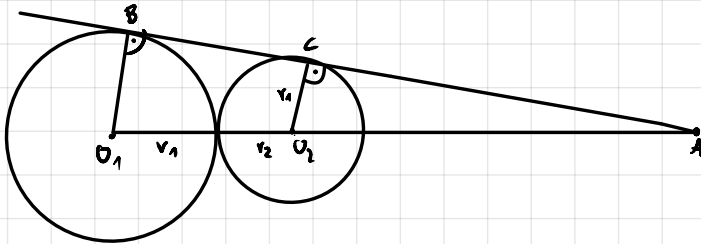
$$a = 2$$

$$r_1 = 3a = 3 \cdot 2 = 6$$

$$r_2 = 4 - a = 4 - 2 = 2$$

$$r_1 = 6 \quad r_2 = 2$$

b)



$$\frac{|O_1 A|}{|O_2 A|} = \frac{|O_1 B|}{|O_2 C|}$$

$$\frac{|O_1 A|}{|O_1 A| - |O_1 O_2|} = \frac{r_1}{r_2}$$

$$\frac{|O_1 A|}{|O_1 A| - 8} = \frac{6}{2}$$

$$\frac{|O_1 A|}{|O_1 A| - 8} = 3$$

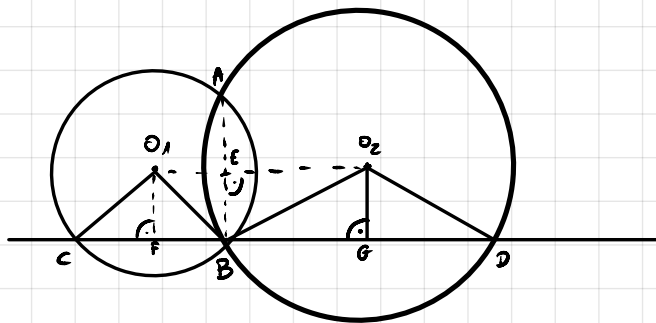
$$|O_1 A| = 3(|O_1 A| - 8)$$

$$|O_1 A| = 3|O_1 A| - 24$$

$$-2|O_1 A| = -24$$

$$|O_1 A| = 12$$

35.



$$|AB| \perp O_1O_2 \wedge AB \perp CD$$

$$AB \parallel FO_2 \wedge AB \parallel GO_2 \wedge CG \parallel O_1O_2$$

$$|O_1E| = |FB| \wedge |O_2E| = |GB| \wedge |O_1F| = |O_2G|$$

$\triangle CBO_1$ i $\triangle BDO_2$ są równoramienne, więc

$$|CF| = |FB| \wedge |BG| = |GD|$$

czyli:

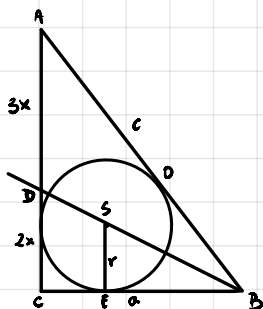
$$|O_1O_2| = |O_1E| + |EO_2| = |FB| + |BG|$$

$$|CD| = |CB| + |BD| = 2|FB| + 2|GD| = 2(|FB| + |GD|)$$

$$2|O_1O_2| = 2(|FB| + |GD|) = |CD|$$

C. N. D.

40.



$$R = \frac{1}{2}c - \text{st. promienia}$$

$$\frac{a}{c} = \frac{2x}{3x} \quad (\text{tn. o podziale boku przez dwusieczną})$$

$$\frac{a}{c} = \frac{2}{3}$$

$$a = \frac{2}{3}c$$

$$(5x)^2 + a^2 = c^2$$

$$25x^2 + a^2 = c^2$$

$$25x^2 + \left(\frac{2}{3}c\right)^2 = c^2$$

$$25x^2 = \frac{5}{9}c^2$$

$$x^2 = \frac{1}{45}c^2 \wedge x > 0 \wedge c > 0$$

$$x = \frac{c}{3\sqrt{5}}$$

$$r = \frac{5x + a - c}{2} = \frac{5 \cdot \frac{c}{3\sqrt{5}} + \frac{2}{3}c - c}{2} = \frac{\frac{5c}{3\sqrt{5}} - \frac{1}{3}c}{2} = \frac{\frac{5\sqrt{5}c}{3 \cdot 5} - \frac{1}{3}c}{2} = \frac{\frac{\sqrt{5}c}{3} - \frac{c}{3}}{2}$$

$$\frac{r}{R} = \frac{\frac{\frac{\sqrt{5}c}{3} - \frac{c}{3}}{2}}{\frac{1}{2}c} = \frac{\frac{\sqrt{5}c}{3} - \frac{c}{3}}{c} = \frac{\frac{\sqrt{5}c}{3} - \frac{c}{3}}{c} = \frac{\sqrt{5} - 1}{3}$$

C. N. D.