9. Ciągi

Zestaw B. Zadania zamknięte

29. $a_n = (-1)^n(25 - 9n^2)$

$a_1 = -(25 - 9) = -16, a_2 = 25 - 9 \cdot 2^2 = -11$

$a_2 - a_1 = -11 + 16 = 5$

Odp. D

CKE **30.** $3n^2 - 25n \leq 0, n \cdot (3n - 25) \leq 0, n \in \left[0; \frac{25}{3}\right]$ oraz $n \geq 1$, zatem $n \in [1; 8]$

Odp. D

CKE **31.** $a_9 = 20 \cdot 10 + 3 = 203$

$b_9 = 2 \cdot 100 - 3 = 197$

$c_9 = 100 + 10 \cdot 10 = 200$

$d_9 = \frac{10 + 187}{10} = 19{,}7$

Odp. B

32. $a_n = \frac{3-6n}{2}, a_{n+1} = \frac{3-6(n+1)}{2} = \frac{-3-6n}{2}$

$r = a_{n+1} - a_n = \frac{-3-6n}{2} - \frac{3-6n}{2} = -3$

Odp. B

33. $a_3 = 24, a_6 = 9$

$a_6 = a_3 + 3r$, więc $r = \frac{1}{3}(a_6 - a_3) = \frac{1}{3}(9 - 24) = -5$

$a_1 = a_3 - 2r = 24 - 2 \cdot (-5) = 34$

$S_5 = \frac{2a_1+4r}{2} \cdot 5 = \frac{2 \cdot 34+4 \cdot (-5)}{2} \cdot 5 = 120$

Odp. B

34. $a_1 = 6, S_5 = 130$

$S_5 = \frac{2a_1+4r}{2} \cdot 5 = \frac{2 \cdot 6+4r}{2} \cdot 5 = (6 + 2r) \cdot 5 = 30 + 10r$

$S_5 = 130$, więc $30 + 10r = 130, r = 10$

$a_4 = a_1 + 3r = 6 + 3 \cdot 10 = 36$

Odp. C

35. $a_7 + a_8 + a_9 = 18$, czyli

$(a_8 - r) + a_8 + (a_8 + r) = 18$

$3a_8 = 18$

$a_8 = 6$

Odp. C

CKE **36.** $a_2 + a_9 = a_4 + a_k$, czyli $2a_1 + 9\mathrm{r} = 2a_1 + 3\mathrm{r} + (\mathrm{k} - 1)\mathrm{r}$, zatem $k = 7$

Odp. B

CKE **37.** $r = 5, a_4 = 3$, zatem $a_1 + a_2 + a_3 + a_4 = 3 + (3 - 5) + (3 - 10) + (3 - 15) = -18$
Odp. C

CKE **38.** $a_3 + a_5 = 58$ zatem: $a_3 + a_5 = a_4 + r + a_4 - r = 58$, czyli $a_4 = 29$
Odp. B

CKE **39.** $a_4 = 2020, a_2 + a_6 = a_4 - 2r + a_4 + 2r = 2 \cdot a_4 = 4040$
Odp. D

40. $a_1 = \sqrt{3}, q = \frac{a_2}{a_1} = \frac{3\sqrt{3}}{\sqrt{3}} = 3$

$a_n = a_1 q^{n-1} = \sqrt{3} \cdot 3^{n-1} = \frac{3^n}{\sqrt{3}}$

Odp. B

CKE **41.** $9a_5 = 4a_3$, czyli $q^2 = \frac{a_5}{a_3} = \frac{4}{9}$ i $q > 0$, czyli $q = \frac{2}{3}$
Odp. A

42. $a_1 = 25, a_2 = x, a_3 = y, a_4 = 5\frac{2}{5} = \frac{27}{5}$

$q^3 = \frac{a_4}{a_1} = \frac{27}{5 \cdot 25} = \frac{27}{125}$

$xy = a_1 q \cdot a_1 q^2 = a_1^2 \cdot q^3 = 25^2 \cdot \frac{27}{125} = 5 \cdot 27 = 135$

Odp. A

43. $a_1 = \sqrt{3} + 1, a_2 = 2, a_3 = 2\sqrt{3} - 2$

$q = \frac{a_3}{a_2} = \frac{2\sqrt{3}-2}{2} = \sqrt{3} - 1$

$a_5 = a_3 q^2 = 2(\sqrt{3} - 1)(\sqrt{3} - 1)^2 = 2(\sqrt{3} - 1)(3 - 2\sqrt{3} + 1) = 2(\sqrt{3} - 1) \cdot 2(2 - \sqrt{3}) =$
$= 4(2\sqrt{3} - 3 - 2 + \sqrt{3}) = 4(3\sqrt{3} - 5) = 12\sqrt{3} - 20$

Odp. B

44. $a_2 = 27, a_5 = 8$

$q^3 = \frac{a_5}{a_2} = \frac{8}{27}$, więc $q = \frac{2}{3}$

$a_4 = \frac{a_5}{q} = \frac{8}{\frac{2}{3}} = 8 \cdot \frac{3}{2} = 12$

Odp. B

CKE **45.** $a_3 = a_1 \cdot a_2$, czyli $a_1 \cdot q^2 = {a_1}^2 \cdot q$, czyli $a_1 = q$
Odp. B

CKE **46.** $\frac{3x}{15} = \frac{5}{9x}$, czyli $9x^2 = 25$ i $x > 0$, więc $x = \frac{5}{3}$
Odp. D

CKE **47.** $a_2 = 6$ i $a_5 = -48$, zatem $q^3 = \frac{-48}{6} = -8$, czyli $q = -2, a_7 = -48 \cdot 4 = -192 < 0$. Ze względu na ujemny iloraz wyrazy a_6 oraz a_8 są dodatnie.
Odp. B

48. $a_1 a_2 a_3 = 729$, więc $\frac{a_2}{q} \cdot a_2 \cdot (a_2 q) = 729$

$a_2^3 = 729$

$a_2 = \sqrt[3]{729} = 9$

Odp. B

CKE

49. $a_n = 2n^2$, zatem $a_5 - a_4 = 50 - 32 = 18$

Odp. D

CKE

50. $a_{n+3} = -2 \cdot 3^{n+1}$, $a_5 = a_{2+3} = -2 \cdot 3^{2+1} = -54$

Odp. A

51. $\begin{cases} a_2 + a_4 = 12 \\ a_6 + a_8 = 52 \end{cases}$, czyli $\begin{cases} (a_1 + r) + (a_1 + 3r) = 12 \\ (a_1 + 5r) + (a_1 + 7r) = 52 \end{cases}$

$\begin{cases} a_1 + 2r = 6 \\ a_1 + 6r = 26 \end{cases}$

$-4r = -20$

$r = 5, a_1 = 6 - 2r = 6 - 2 \cdot 5 = -4$

$a_n = a_1 + (n-1)r = -4 + (n-1) \cdot 5 = 5n - 9$

A. $r > 0$, więc ciąg (a_n) jest rosnący.

B. $a_1 < 0$

C. $a_n = 5n - 9$

D. Żaden wyraz ciągu nie jest podzielny przez 5.

E. $S_{20} = \frac{2a_1 + 19r}{2} \cdot 20 = \frac{2 \cdot (-4) + 19 \cdot 5}{2} \cdot 20 = 87 \cdot 10 = 870$

F. $a_n < 0$, gdy $5n - 9 < 0$, $n < \frac{9}{5}$, $n = 1$, czyli ciąg (a_n) ma dokładnie jeden wyraz ujemny.

Odp. CE

52. $a_1 = 13, a_{10} = 31$

$a_{10} = a_1 + 9r$, więc $r = \frac{1}{9}(a_{10} - a_1) = \frac{1}{9}(31 - 13) = \frac{1}{9} \cdot 18 = 2$

$S_{30} = \frac{2a_1 + 29r}{2} \cdot 30 = (2 \cdot 13 + 29 \cdot 2) \cdot 15 = 1260$

$a_n = 13 + (n-1) \cdot 2 = 2n + 11$

Odp. PP

53. $a_1 = 48, a_2 = 36, a_3 = x - 2$

$q = \frac{a_2}{a_1} = \frac{36}{48} = \frac{3}{4}$

$a_3 = a_2 q = 36 \cdot \frac{3}{4} = 27$, więc $x - 2 = 27$, $x = 29$

$a_5 = a_3 q^2 = 27 \cdot \left(\frac{3}{4}\right)^2 \notin \mathbb{N}$

Odp. FF

54. $a_1 = 5$ i $S_6 = 300$

$S_6 = \frac{2a_1 + 5r}{2} \cdot 6 = (2 \cdot 5 + 5r) \cdot 3 = 30 + 15r$

$S_6 = 300$, więc $30 + 15r = 300$

$15r = 270$

$r = 18$

$a_{12} = a_1 + 11r = 5 + 11 \cdot 18$ – to nie jest liczba podzielna przez 3.

Odp. PF

55. $a_n = -\frac{3}{4^n}, a_1 = -\frac{3}{4} < 0$

$q = \frac{a_{n+1}}{a_n} = -\frac{3}{4^{n+1}} : \left(-\frac{3}{4^n}\right) = \frac{3}{4^{n+1}} \cdot \frac{4^n}{3} = \frac{1}{4} \in (0; 1)$

$a_1 < 0$ i $q \in (0; 1)$, więc ciąg (a_n) jest rosnący.

Odp. B2

CKE

56. $a_n = n^2 - n$, $a_{n+1} = (n+1)^2 - \mathrm{n} - 1 = n^2 + \mathrm{n}$, zatem $a_{n+1} - a_n = 2n > 0$, czyli ciąg jest rosnący.

Odp. A3