①

$$\lim_{n\to\infty}\left(\frac{9n^3+11n^2}{7n^3+5n^2+3n+1}-\frac{n^2}{3n^2+1}\right)=\lim_{n\to\infty}\frac{9n^3+11n^2}{7n^3+5n^2+3n+1}-\lim_{n\to\infty}\frac{n^2}{3n^2+1}=$$

$$=\lim_{n\to\infty}\frac{\cancel{n^3}\left(9+\frac{11}{n}\right)}{\cancel{n^3}\left(7+\frac{5}{n}+\frac{3}{n^2}+\frac{1}{n^3}\right)}-\lim_{n\to\infty}\frac{\cancel{n^2}}{\cancel{n^2}\left(3+\frac{1}{n^2}\right)}=\frac{9}{7}-\frac{1}{3}=\frac{20}{21}=\boxed{0{,}952\ldots}$$

$\frac{11}{n}\to 0$, $\frac{5}{n}\to 0$, $\frac{3}{n^2}\to 0$, $\frac{1}{n^3}\to 0$, $\frac{1}{n^2}\to 0$

② $(m^2-m)x^2+2m^2x+m^2-4=0 \quad (*)$

1°) Równanie (*) jest równaniem kwadratowym $\Leftrightarrow$ gdy:

$a\neq 0$

$m^2-m\neq 0$

$m(m-1)\neq 0$

$m\neq 0 \ \wedge \ m\neq 1$

$\Rightarrow$ Równanie (*) jest równaniem kw. $\Leftrightarrow$ gdy $m\in R\setminus\{0,1\}$.

2°) Równanie kwadratowe ma dwa różne rozw. $\Leftrightarrow$ gdy:

$\Delta>0$

$4m^4-4\cdot(m^2-m)\cdot(m^2-4)>0$

$4m^4(-4m^2+4m)(m^2-4)>0$

$\cancel{4m^4}-\cancel{4m^4}+16m^2+4m^3-16m>0 \quad /:4$

$4m^3+16m^2-16m>0 \quad /:4$

$m^3+4m^2-4m>0$

$m(m^2+4m-4)>0$

$m=0 \quad \vee \quad m^2+4m-4=0$

$m=0 \quad \vee \quad m=-2+2\sqrt{2} \ \vee \ m=-2-2\sqrt{2}$

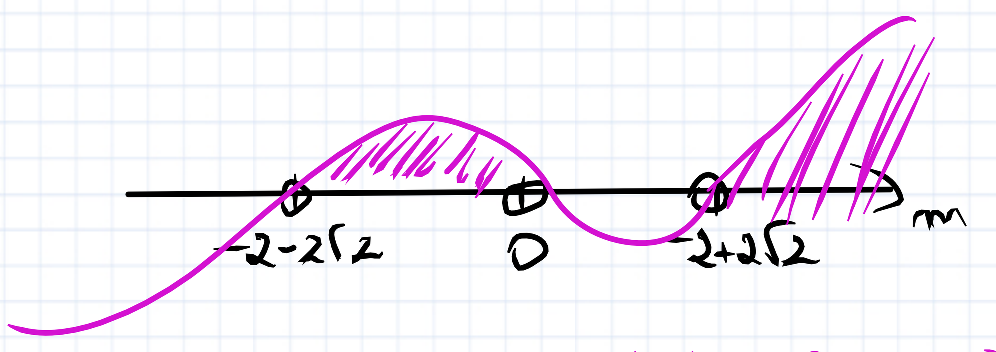
$m^2+4m-4=0$

$\Delta=16-4\cdot1\cdot(-4)=32$

$\sqrt{\Delta}=\sqrt{32}=4\sqrt{2}$

$m_1=\frac{-4+4\sqrt{2}}{2}=-2+2\sqrt{2}$

$m_2=-2-2\sqrt{2}$



$\Rightarrow m\in(-2-2\sqrt{2};0)\cup(-2+2\sqrt{2};+\infty)$

3°) Warunek z zadania:

$\frac{1}{1+x_1}+\frac{1}{1+x_2}\leqslant \log_{343}49$

$\frac{1+x_2+1+x_1}{(1+x_1)(1+x_2)}\leqslant\frac{2}{3}$

$\frac{x_1+x_2+2}{1+x_2+x_1+x_1x_2}-\frac{2}{3}\leqslant 0$

$\frac{-\frac{b}{a}+2}{1+\left(\frac{-b}{a}\right)+\frac{c}{a}}-\frac{2}{3}\leqslant 0$

$\frac{-\frac{b}{a}+2}{\frac{a-b+c}{a}}-\frac{2}{3}\leqslant 0$

$\frac{-b+2a}{\cancel{a}}\cdot\frac{\cancel{a}}{a-b+c}-\frac{2}{3}\leqslant 0$

$\frac{2a-b}{a+c-b}-\frac{2}{3}\leqslant 0$

$\frac{3(2a-b)-2(a+c-b)}{3(a+c-b)}\leqslant 0$

$\frac{6a-3b-2a-2c+2b}{3(a+c-b)}\leqslant 0$

$\frac{4a-b-2c}{3(a+c-b)}\leqslant 0$

$\frac{4\cdot(m^2-m)-2m^2-2m^2+8}{3(\cancel{m^2}-m+\cancel{m^2}-4-\cancel{2m^2})}\leqslant 0$

$\frac{\cancel{4m^2}-4m-\cancel{4m^2}+8}{3(-m-4)}\leqslant 0$

$\frac{-(4m-8)}{-(3m+12)}\leqslant 0$

$(4m-8)(3m+12)\leqslant 0$

$4m-8=0 \quad \vee \quad 3m+12=0$

$4m=8 \quad \vee \quad 3m=-12$

$m=2 \quad \vee \quad m=-4$

Korzystam z wzorów Viete'a:

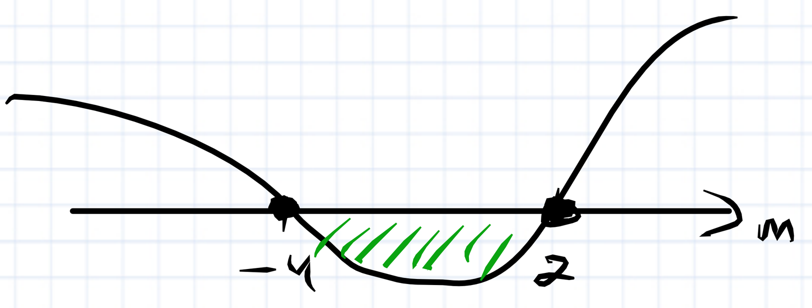
$x_1x_2=\frac{c}{a}$

$x_1+x_2=-\frac{b}{a}$

Dodatkowe założenie:

$3m+12\neq 0$

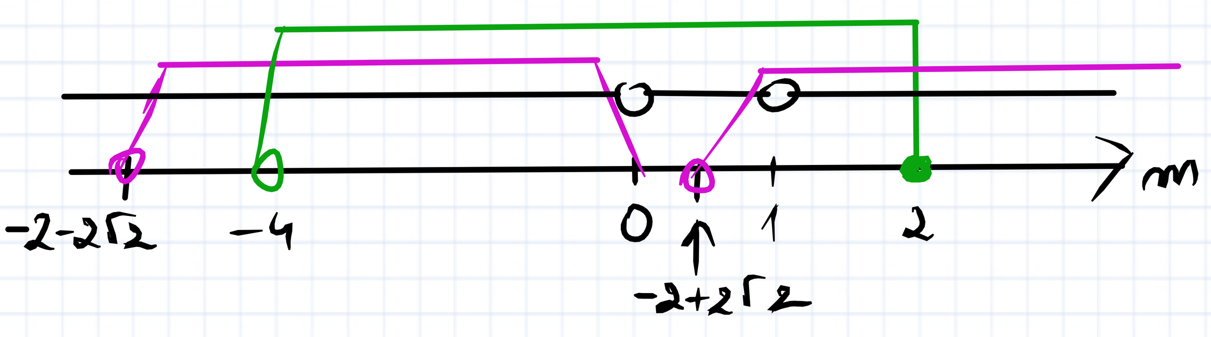
$m\neq -4$



$\Rightarrow m\in(-4,2\rangle$

4°) Odpowiedź:

$$\begin{matrix}1^\circ\\2^\circ\\3^\circ\end{matrix}\begin{cases}a\neq 0\\ \Delta>0\\ \frac{1}{x_1+1}+\frac{1}{x_2+1}\leqslant\log_{343}49\end{cases}\Rightarrow\begin{cases}m\in R\setminus\{0,1\}\\ m\in(-2-2\sqrt{2};0)\cup(-2+2\sqrt{2};+\infty)\\ m\in(-4,2\rangle\end{cases}$$



$\Rightarrow m\in(-4;0)\cup(-2+2\sqrt{2};2\rangle$

m które są rozwiązaniem i należą to zbioru liczby całkowitych:

$\Rightarrow m\in\{-3,-2,-1,2\}$